

Order of Operations

If we have $5 \times 7 + 2 - 4 \times 3 + (1 + 2^2)^3$, what is the answer???
How do we do this???

We need some convention, so we know what to do first and what to do second, so we can all get the same answer.

What is the convention?

The order is:

1. Parentheses first;
2. then Exponents;
3. then, from left to right, all the Multiplication and Division;
4. finally, from left to right, all the Addition and Subtraction.

Remember "Please Excuse My Dear Aunt Sally." Or: "PEMDAS."

Please	Excuse	My Dear	Aunt Sally
Parentheses	Exponents	Multiplication-Division	Addition-Subtraction

Examples.

$$\begin{aligned} 1. \quad 10 - 7 - 1 + 18 + 3 &= 3 - 1 + 18 + 3 \\ &= 2 + 18 + 3 \\ &= 20 + 3 \\ &= 23 \end{aligned}$$

Remember: do addition and subtraction as you find them from left to right. If we had to do all the addition first, we could never have a bank account!! We'd have to put all our money in before we ever took any out!! But reality does not work that way: we put money in and take it out day after day, week after week, year after year.

$$\begin{aligned}
2. \quad 40 \div 8 \times 6 \times 4 \div 5 &= 5 \times 6 \times 4 \div 5 \\
&= 30 \times 4 \div 5 \\
&= 120 \div 5 \\
&= 24
\end{aligned}$$

$$\begin{aligned}
3. \quad 54 + 19 - 5 - 29 + 3 &= 73 - 5 - 29 + 3 \\
&= 68 - 29 + 3 \\
&= 39 + 3 \\
&= 42
\end{aligned}$$

$$\begin{aligned}
4. \quad 20 \times 4 \div 5 \times 10 \div 2 &= 80 \div 5 \times 10 \div 2 \\
&= 16 \times 10 \div 2 \\
&= 160 \div 2 \\
&= 80
\end{aligned}$$

$$5. \quad 3 \times (50 - 14) \times 2 \div (2 + 2 \times 5)$$

Do what is in parentheses first!

In one parentheses, we have $50 - 14$. This is 36.

In the other parentheses, we have $2 + 2 \times 5$. We need to do PEMDAS on this. So, we'd multiply before we add:

do 2×5 before adding 2!

$$2 + 2 \times 5 = 2 + 10 = 12$$

$$\begin{aligned}
\text{Then } 3 \times (50 - 14) \times 2 \div (2 + 2 \times 5) &= 3 \times 36 \times 2 \div 12 \\
&= 108 \times 2 \div 12 \\
&= 216 \div 12 \\
&= 18
\end{aligned}$$

$$\begin{aligned}
6. \quad 3 \times 50 - 14 \times 2 \div 2 + 2 \times 5 &= 150 - 14 \times 2 \div 2 + 2 \times 5 \\
&= 150 - 28 \div 2 + 2 \times 5 \\
&= 150 - 14 + 2 \times 5 \\
&= 150 - 14 + 10 \\
&= 136 + 10 \\
&= 146
\end{aligned}$$

When we know this stuff better, we can do all the multiplication and division between each addition or subtraction sign:

$$\begin{aligned}
&3 \times 50 - 14 \times 2 \div 2 + 2 \times 5 \\
&= 150 - 14 + 10 \\
&= 136 + 10 \\
&= 146
\end{aligned}$$

Remember: we multiply and divide before we add and subtract. If you have \$98,423 in the bank, and you deposit four \$500 checks, do we do \$98,423 + \$500, then multiply that by 4? No! That would be silly! We multiply \$500 by 4, then add that to \$98,423. So,

$$\$98,423 + \$500 \times 4 = \$98,423 + \$2,000 = \$100,423.$$

Reality says to multiply and divide, from left to right, before you do any adding or subtracting.

$$\begin{aligned}
7. \quad 90 - 3 \times 5 + 18 - 2 \times 30 + 45 &= 90 - 15 + 18 - 2 \times 30 + 45 \\
&= 90 - 15 + 18 - 60 + 45 \\
&= 75 + 18 - 60 + 45 \\
&= 93 - 60 + 45 \\
&= 33 + 45 \\
&= 78
\end{aligned}$$

When we know this stuff better, we can do all the multiplication and division between each addition or subtraction sign:

$$\begin{aligned} & 90 - 3 \times 5 + 18 - 2 \times 30 + 45 \\ = & 90 - 15 + 18 - 60 + 45 \\ = & \quad 75 + 18 - 60 + 45 \\ = & \quad \quad 93 - 60 + 45 \\ = & 78 \end{aligned}$$

$$\begin{aligned} 8. \quad 5 \times 19 - 2 \times 14 + 40 \div 5 &= 95 - 2 \times 14 + 40 \div 5 \\ &= 95 - 28 + 40 \div 5 \\ &= 95 - 28 + 8 \\ &= \quad \quad 67 + 8 \\ &= 75 \end{aligned}$$

When we know this stuff better, we can do all the multiplication and division between each addition or subtraction sign:

$$\begin{aligned} & 5 \times 19 - 2 \times 14 + 40 \div 5 \\ = & 95 - 28 + 8 \\ = & \quad 67 + 8 \\ = & 75 \end{aligned}$$

$$9. \quad (3^2 + 1)^2 \times 5 + (19 \times 2 - 7 \times 5)^2$$

Do what is in parentheses first!

In one parentheses, we have $3^2 + 1$.

We need to do the exponent before the addition:

$$3^2 + 1 = 9 + 1 = 10$$

In the other parentheses, we have $19 \times 2 - 7 \times 5$.

We need to multiply before subtracting:

$$19 \times 2 - 7 \times 5 = 38 - 35 = 3$$

$$\begin{aligned} \text{So we have } (3^2 + 1)^2 \times 5 + (19 \times 2 - 7 \times 5)^2 &= 10^2 \times 5 + 3^2 \\ &= 100 \times 5 + 9 \\ &= 500 + 9 \\ &= 509 \end{aligned}$$

10. $5 \times 7 + 2 - 4 \times 3 + (1 + 2^2)^3$

Do what is in parentheses first!

In the parentheses, we have $1 + 2^2$.

We need to do the exponent before the addition:

$$1 + 2^2 = 1 + 4 = 5$$

Then we get $5 \times 7 + 2 - 4 \times 3 + 5^3$.

Do the exponent next! $5 \times 7 + 2 - 4 \times 3 + 125$

Then do all the multiplication and division.

$$5 \times 7 + 2 - 4 \times 3 + 125 = 35 + 2 - 12 + 125$$

Then do all the addition and subtraction.

$$\begin{aligned} 35 + 2 - 12 + 125 &= 37 - 12 + 125 \\ &= 25 + 125 \\ &= 150 \end{aligned}$$

Exercises

1. $41 + 56 - 19 + 8 - 25$
2. $\$1800 + \$5200 - \$390 - \$1200 + \$1800$
3. $32 \div 8 \times 12 \times 4 \div 2$
4. $4 \times 30 \times 5 \div 10 \times 7 \div 6$
5. $6 \times 5 + 4 - 19 + 100 \div 4$
6. $100 \div 4 \times 3 + 18 \times 3 - 50 \div 2 \div 5$
7. $(4 \times 3 + 1) \times 3 - 5 \times 6$
8. $2 \times 3 \times 3 - (5 \times 5 - 19) + 3 \div 4$
9. $3 \times 13 - 5 + 4 \times 4 + 1$
10. $3 \times (13 - 5 + 4) \times 4 + 1$
11. $3 \times (13 - 5 + 4 \times 4) + 1$
12. $3 \times (13 - 5) + 4 \times (4 + 1)$
13. $4^2 + 11 \times 34 - 6^2 \div 2$
14. $19 \times 2^2 + 18 \div 3^2 - 12 \div 2^2$
15. $3 \times (13 - 5)^2 + 4 \times (4 + 1)$
16. $3 \times (13^2 - 5) + 4 \times (4^2 + 1)$

Answers

1. 61

2. \$7,210

3. 96

4. 70

5. 40

6. 124

7. 9

8. 12.75

9. 51

10. 145

11. 73

12. 44

13. 372

14. 75

15. 212

16. 560